Course title: **DISCRETE STRUCTURES WITH GRAPH THEORY**

Lecturers	Full Prof. Blaženka Divjak, Ph.D. Asst. Prof. Marcel Maretić, Ph.D. Damir Horvat, M.A.
Language of instruction	Croatian and English
Study level	Master
Study programme	Information and Software Engineering / Business Systems Organization
Semester	1st (winter)
ECTS	6
Goal	The main goal of the course is to introduce the students to the mathematical theories necessary for information sciences (discrete mathematics, graph theory). One of the goals of this course is to support the students' developing skills of rigorous mathematical thought, necessary for a successful career in ICT. Students should become familiar with different formalisms. Students should be able to observe connections between formal theory and real-world situation. This course aids to development of mathematical strictness (to a certain degree) and facilitates students to adequately apply formalisms to specific problem situation. Additionally, the goal is to develop connections between discrete mathematics, informatics and algorithmic thinking specially related to cryptography and using graphs to solve problems in ICT.
General and specific learning outcomes	
Content	**1. Models and structure of mathematics (2 hours)** Mathematical models. Characteristics of mathematical models. Structure of mathematics. Role of axioms in mathematical theory. Basic and derived notions, relations between notions. Theorems. Examples and evolution of well known mathematical theories. **2. Methods of proving statements in mathematics (2 hours)** Propositional and predicate logic. Propositions and operations with propositions. Complex expressions related to AND, OR. Implication and its characteristics. Structure of statement. Techniques of mathematical proof: direct proof, proof by cases, proof by contraposition, proof of (several) equivalent statements, counterexamples. Mathematical induction as instrument of proving on natural (integer) numbers. Peano's axioms. **3. Relations (4 hours)** Set as the basic mathematical notion. Paradoxes of the set theory. Cantor's and Zermelo's to the set theory. The power set. Operations on sets (union, intersection, set difference, complement of a set) and their properties. Venn's diagrams. Problems with discrete sets and continuum. Cartesian products of sets (discrete and continuous sets). Sets of numbers. Complex numbers. Relations, binary relations, properties of binary relations. Matrix of incidence. Functions (domain, range, bijection, countability of sets). Inverses functions and composition of functions. **4. Congruences with applications (2 hours)**

	Equivalence relation. Quotient set of an equivalence relation. Set partition induced by equivalence relation. Congruences. Congruence operations (addition, subtraction, multiplication). Solving congruences. Chinese remainder theorem. Determining numbers through residues. Applications of congruences in codes and crypotography (International Standard Book Number, Universal product Code, cyphers). **5. Well ordered sets and lattices (4 hours)** Partial order on a set, definition and basic examples. Lexicographic order. Comparability of elements in partially ordered set. Hasse's diagrams and their implementation. Linearly or totally ordered set. Notion of minimal and maximal, least upper and greater lower bound in partially ordered set. Well ordered set. Lattice: definition and examples. Divisibility of integer numbers as relation of partial order. The largest common divisor and Euclidean algorithm. The least common multiple. The lattice of divisors of a natural numbers. Well ordering principle (any nonempty set of natural numbers has a smallest element). Equivalence of principle of mathematic induction and well ordering principle on a set. Prime numbers and their characteristics. The sieve of Eratosthenes. The fundamental theorem of arithmetic (every integer number larger than 2 can be written as a product of powers of distinct prime numbers). The Goldbach's conjecture. **6. Graphs (2 hours)** Definition of graph and basic characteristics of graphs. Degree of vertex, multiple edges, pseudograph. Subgraph. Special graphs: complete graph, bipartite graph, and complete bipartite graph. Regular graphs. Euler's proposition (the sum of the degrees of the vertices is an even number equal to twice the number of edges). Number of odd vertices is even. Isomorphism of graphs. Connection of isomorphic graphs over permutation matrix. Invariants of isomorphic graphs. Walk, closed walk, path in graph. **7. Paths and cycles (4 hours)** Eulerian circuit as closed Eulerian path and Eulerian graph. Connected graphs. A graph is Eulerian if and only if it is connected and every vertex is even. Solution to the problem of Koeningsberg's bridges. Hamiltonian cycle. Hamiltonian graph. The Petersen graph as a counterexample to Hamiltonian graph. Open problem of necessary and sufficient condition for Hamiltonian graph. Incidence matrix and adjacency matrix. Characteristics of adjacency matrix. Algorithm for finding Eulerian path and cycle. Problem of deadlocks. **8. Weighted graph. Shortest path problem. (2 hours)** Weighted graph. Shortest path between two vertices in a weighted graph. Graph application. The traveling salesman's problem. Dijkstra's algorithm and improved Dijkstra's algorithm. The Floyd-Warshall algorithm. Comparison of these two algorithms, their complexity and possible usage. The Chinese postman problem in Eulerian graph and in graph which is not Eulerian. "Eulerization" of graph. **9. Trees (2 hours)** Tree as a connected graph which does not contain any cycles. Properties of a tree. Forest. Rooted tree. Binary tree. Binary tree searching. Sorting algorithm. Binary tree in presentation of algebraic identity. Minimal spanning tree. Algorithms for minimal spanning tree. Notion of subgraph. Spanning subgraph. Weight of tree. Problem of finding minimal spanning tree. Minimal spanning tree algorithms. Kruskal's algorithm. Prim's algorithm. Tree searching. Passing through graph and tree.

	10. Directed graphs (4 hours) Directed graph. Directed trail, directed path. Tournament: definition and features. Existence of directed Hamiltonian path in every tournament. Lattices and critical path. Problem of distribution. Critical path method - CPM, PERT. Applications in project planning and management. Transported network. Value of a flow. Notion of f-augmenting path. Flows and cuts. Max flow – min cut theorem. Proving theorem. Examples of application. **11. Graph colouring (2 hours)** Problem of four colours. Historical review and Appel-Haken solution. Graph vertex colouring. Chromatic number of graphs and examples. Applications of chromatic number on schedule problems. Cliques and number of cliques. Colouring of graph edges. Chromatic number theorems. Chromatic numbers of some known graphs. Theorem (Vizig, Gupta). Matching in graphs. Maximum matching in graph. Matching in bipartite graphs. Hall's theorem.
Exercises	Seminars follow lectures At the seminars, exercises related to the theory discussed in the lectures are addressed. Emphasis is, wherever possible, on practical exercises that may be related to some realistic problem. It also shows how some exercises can be solved in some programming tools or languages such as Python, SAGE, and Maxima. Students also solve two practical exercises in Python (or SAGE, or Maximi) One exercise pertains to discrete mathematics and is mainly related to some application in the theory of numbers and combinatorial, while the other task is related to the theory of graphs.
Realization and examination	Continuous assessment of students' learning is done throughout the semester. Elements for assessment: 3 monthly tests; homework in e-learning system; exercises and simple problem solving in classrooms (quizzes); project work in teams on project posing and project solving. If a student doesn't satisfy criteria during semester, she/he will be assessed by written and oral examination.
Related courses	
Literature	Primary: Goodaire E. G., Parmenter M. M., Discrete Mathematics with Graph Theory, Prentice Hall, New York, 2002. Additional: Divjak B., Lovrenčić A., Diskretnamatematika s teorijom grafova, TIVA-FOI, Varaždin, 2005. Garnier R., Taylor J., Discrete Mathematics for New Technology, Institute of Physics Publishing, Bristol & Philadelphia, 1999. Veljan D.: Konačnamatematika s teorijomgrafova, Algoritam 2003 Tucker, A.: Applied Combinatorics, John Wiley & Sons, New York, 1995 Sedgewick, R.: Algorithms in C++, Part 5: Graph Algorithms, Addison-Wesley, Boston, 2002. Barwise, J., Etchemendy, J.: The Language of First-Order Logic, CSLI, Stanford, 1992.

	Red. Nečepirenko, M. I.: Algoritmy I programmyrešenijazadačnagrafahisetjah, Nauka, Novosibirsk, 1990. Knuth, D. E.: The Art of Computer Programming: sorting and Searching, Addison-Weslay, Reding, 1973 Garey, M. R., Johnson, D. S.: Computers and Intractability, W. H. Freeman & Co., New York, 1979.